[1] noraspinto@yahoo.co.uk

## ASSESSING THE RELIABILITY OF DUAL FREQUENCY DIFFERENTIAL GNSS OBSERVATIONS FOR ORTHOMETRIC HEIGHT DETERMINATION IN PARTS OF ACCRA, GHANA.

Otoo-Kwofie ,C.

Department of Civil Engineering, Cape Coast Polytechnic, P. O. Box AD 50, Cape Coast

**ABSTRACT**
Height refers to the vertical distance of a point above or below a reference datum. Ordinary Level offers accurate height values, but its use is time-consuming, labor intensive and limited in sighting distances. The GNSS technology is making such traditional methods of surveying unpopular in Ghana. This study uses simple linear regression analysis, root mean square errors, calculation of residuals and graphs to assess the reliability of differential GPS/GLONASS observations made with dual frequency receivers and compared with MSL heights determined trigonometrically.  The study area covers the major roads in Accra. The road sections with a total chainage of about 13km includes a section of George Walker Bush Motorway (N1), Olusegun Obasanjo Highway and a section of the Liberation road.The Trimble combined GNSS receivers (R4) and the Sokkia 1'' Total Station (Set 310) were used for the data collection. The Geoidal separations (undulations) for the study area were estimated using Alltrans EGM2008 Calculator 1.2.  The analysis showed that the combined GNSS derived Orthometric heights are reliable and comparable to the MSL heights determined trigonometrically. The arithmetic residuals between the derived Orthometric heights and MSL heights ranged between -0.5025m to +1.4365m with an average residual of +0.5730m. The root mean square error (RMSE) of the observation was estimated as 0.7198m, while the Correlation Coefficient was 0.9981; denoting a measure of higher quality of fitbetween the GNSS derived orthometric heights and those of MSL from Trigonometry. The average least squares residual from the regression analysis was estimated in absolute terms, to be 0.5970m. This accuracy in height can be used to produce contour maps at a smaller scale in Ghana with respect to the national survey and mapping specifications. It is recommended that, surveyors and engineers in Ghana adopt fully, GNSS receivers in height determination. In addition to this, the determination of the National Geoid Model by Ghana Land Administration project should be seriously pursued since it is a major pre-requisite for sustainable height realization in Ghana.
**Keywords**: Levelling, GNSS, GPS, GLONASS, Geoid, Ellipsoid, Orthometric.

## 1.0  INTRODUCTION

Levelling refers to the process of determining the height of points above or below a reference surface by measuring directly or indirectly the difference in elevation between two or more points (Mohammed, et al; 2012). The heights of points relative to a chosen surface are known as reduced levels of these points, and the reference surfaces are usually known as a datum.
Levelling has many applications. In engineering, it is used in all stages of construction projects from the initial site survey through to the final setting out (Uren and Price, 1985). The fieldwork can be carried out using different survey equipment, procedures and techniques. The field techniques and procedures may be classified as precise, when precise levels (either optical or digital levels) are used, or it may be referred to as ordinary, when using ordinary levels (ordinary optical, automatic or digital levels). Moreover it can be carried out trigonometrically or barometrically. It can also be further classified as ordinary or

geodetic depending on the length of the lines involved and the area of the earth surface covered by the operation.

In ordinary levelling, short distances or lines are involved and small area of the earth surface is covered by the operations (Bannister and Raymond, 1995). On the other hand,Geodetic levelling involves height determinations where large areas of the earth's surfaces are covered and therefore longer lines or distances are involved. In such projects due to the longer lines of level, the curvature of the earth surface, atmospheric refraction and the geoid undulation of the earth have to be taken into account. In such circumstances, direct or ordinary levelling becomes limited with the whole levelling process becoming more complicated and complex, and the use of ordinary levels to achieve the first order accuracy requirements and precisions expected becoming practically impossible.

Heights can be measured using various devices such as the total stations, handheld lasers, GPS devices and the level instrument (Uren and Price, 2006). Geometric Levelling, which is the traditional technique of determining heights using the level instrument, offers an inexpensive, simple and accurate method for measuring heights; specifically, reliable and precise as a vertical displacement measurement method (Henriques and Casaca, 2001). Most importantly, it should be noted that in this techniques of levelling the surface of measurement which is the earth is assumed to be plane (Uren and Price, 2006). However, as the size of the survey or the construction project increases, it is no longer feasible to assume a plane in defining positions because the effect of the earth's curvature and the undulating nature of the terrain become too large to ignore. This has resulted in different types of local heights systems globally which can be categorised into two types depending on the reference datum and the technique adopted in determining the height. These are Mean Sea level (MSL) or Orthometric heights and Geodetic or Ellipsoidal heights.
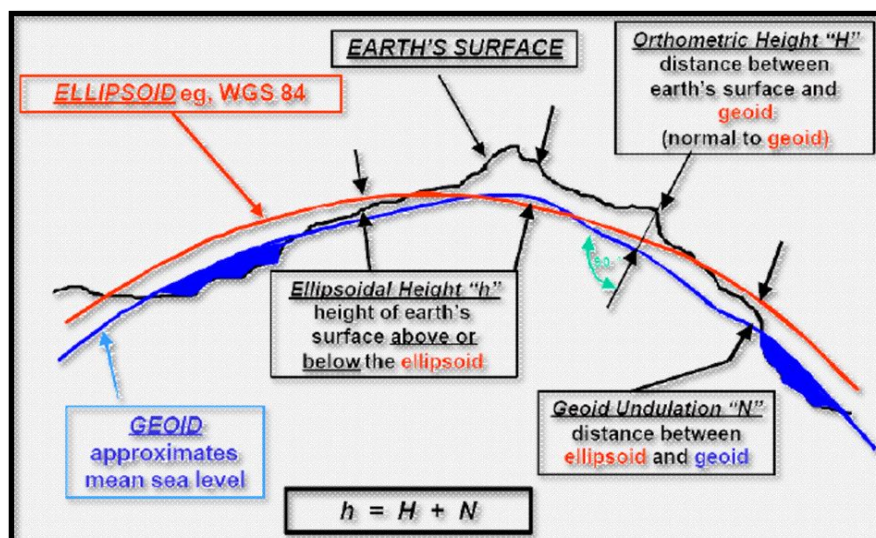


**Figure 1: The Geoid-Ellipsoid Diagram, (source: www.nptel.ac.in).**

Officially, the earth gravitational model(EGM) 2008 was publicly released by the U.S. National Geospatial-Intelligence Agency (NGA), EGM Development Team (Mohammed, et al. 2012) and is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159 (Abd-Elmotaal, 2008). This degree and order of spherical harmonics describes the complexity of the polynomial defining the reference geo-potential model. The model's gravitational potential is defined below (Abd-Elmotaal, 2008):

$$V(r,\theta,\lambda) = \frac{GM}{r}\left[1 + \sum_{n=2}^{\infty}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}(C_{nm}\,\cos m\lambda + S_{nm}\,\sin m\lambda)P_{nm}\,(\cos\theta)\right]$$

where

| | |
|---|---|
| GM | = the geocentric gravitational constant |
| r | = the geocentric radius, |
| $\theta$ | = the polar distance, |
| $\lambda$ | = the geodetic longitude, |
| $a$ | = the equatorial radius of the mean earth's ellipsoid, |
| $P_{nm}$ | = the associated fully normalized Legendre functions |
| $C_{nm}$ and $S_{nm}$ | = the fully normalized potential coefficients. |

The WGS 84 constants used to define the reference ellipsoid, and the associated normal gravity field, to which the geoid undulations are referenced are:
$a$ = 6378137.00m (semi-major axis of WGS 84 ellipsoid),
$f$ = 1/298.257223563 (flattening of WGS 84 ellipsoid),
$GM$ = 3.986004418 x $10^{14}$ $m^3s^{-2}$ (Product of the Earth's mass and the Gravitational Constant),
$\omega$ = 7292115 x $10^{-11}$ radians/sec (Earth's angular velocity) (Mohammed, et al. 2012).

The harmonic synthesis software applies a constant, zero-degree term of - 41 cm to all geoid undulations computed using EGM2008 with the height anomaly to geoid undulation correction model (also provided). Similarly, all pre-computed geoid undulations incorporate this constant zero-degree term. This term converts geoid undulations that are intrinsically referenced to an ideal mean-earth ellipsoid into undulations that are referenced to WGS 84.
In Ghana, currently the position of points are defined in geographic terms based on the geodetic datum British War Office Spheroid, and the horizontal Cartesian system is based on a modified Transverse Mercator Projection (Ayer and Ofosu, 2006).This places the whole country at the same origin of the intersection of longitude 1° W and latitude 4° 40' N, and the values assigned this origin is 0.000ft for northings and 900,000.000ft for eastings with a scalar factor of 0.99975 along the central meridian (Ayer and Ofosu, 2008). The central meridian to which all bearings are referred is the meridian 1° W and the direction of its plane

[1] noraspinto@yahoo.co.uk

is taken as the northings axis, and the plane of the parallel latitude 4° 40' N forms the eastings axis (Survey Records, 1936).

Heights in Ghana are based on the Mean Sea Level (MSL) determined in Accra by means of tidal observations conducted from April 9, 1922 to April 30, 1923 (Survey Records, 1936).

## 1.1.    Problem Statement

Nowadays the direct height determination by ordinary leveling has become unpopular and virtually non-recommendable in precise vertical control surveys. This may be due to its tedious nature, delay and time-consuming field operations, labour intensive, limitation in sighting distances, etc. In addition to this, the emergence of the GNSS technology in positioning has totally discouraged the use of ordinary levels in height determination in Ghana.

Nevertheless, Geomatics professionals in Ghana are more conversant with, and by experience more knowledgeable in and trust the use of GPS receivers in horizontal positioning than its use in height determination most especially orthometric heights which are referenced to the geoid. This may be because of lack of knowledge on the basic concepts and the field technicalities involved in such operations to produce enhanced results, conversion from ellipsoidal heights to orthometric heights, and lack of trust in the reliability of the end results.

This paper looks into the combined GNSS differential observations in height determination using a dual frequency GNSS receiver and thereof, assesses the reliability of the height information.

## 1.2 Aim and Objectives of The Study

This study seeks to evaluate statistically the reliability of Orthometric heights determined using GNSS observations.

The specific objectives of the study are to:

1. Establish vertical monuments/controls and determine their Orthometric heights by differential leveling using Total Station equipment.
2. Determine the ellipsoidal heights of the vertical monuments using a dual frequency combined GNSS(GPS/GLONASS) receiver by differential techniques.
3. Determine the residuals and root mean square errors (RMSE) of the results, as a way of estimating accuracy.
4. Compare the difference(s) between the two set of height values using Linear Regression Analysis.

## 1.3 STUDY AREA

The study area is a section of Greater Accra with a population of about 3.9 million and is the capital city of Ghana (Ghana Statistical Service, 2010 Population and Housing Census). The major routes of concern were a section of George Walker Bush Motorway (N1), Olusegun Obasanjo Highway and a section of the Liberation road as indicated in blue ink shown in figure 2.

[1] noraspinto@yahoo.co.uk

Figure 2: Satellite Image of Part of Accra (source: Images ©2013 Terra Metric, Map data ©2013 Google Maps)

### 1.4 Justification of The Study

Differential Positioning using dual frequencies receivers helps to improve the data quality considerably by refining inaccuracies and errors associated with absolute positioning and ambiguities in the use of single frequency receivers. Over the years, the GNSS receivers used in determining positions in Ghana, operate with the United States' GPS system. Nowadays, the mapping sector is witnessing the advent of GNSS receivers built to operate in the combined Russian' GLONASS (GLObal'nayaNavigatsionnayaSputnikovayaSistema) system and United States' GPS. This implies dual frequency receivers coupled with the combined GNSS (GPS/GLONASS) technology would help evaluate optimally the reliability of this satellite technique of height determination.

Following current and future trends, with the emergence of multiple GNSS, constellation-specific receivers and multiple constellations, GNSS receivers are increasingly being produced and are in the market. It is therefore important for users to gain maximum benefit from the GNSS receivers, system and signal even in the area of height determinations, by way of knowledge supported by empirical evidence, as to how to produce quality, reliable and good Orthometric height information.

Finally, this knowledge would improve Orthometric height determination in Ghana by making it easier, cheaper, speedy and less time-consuming, cost-effective, less labour-intensive and almost automated with no limitation in sight distances.

## 2.0 METHODOLOGY

### 2.1 Secondary Data Sets Used

- The Ghana National Coordinate Values (X, Y, Z) of the existing survey controls around the area.
- Geoid-ellipsoid separation (undulation) extracted from EGM2008 geodetic model calculator (i.e. Alltrans EGM2008 Calculator 1.2 of resolution 10'*10') using the observed WGS84 geographical coordinates.

### 2.1.1 Primary Data Collection

The primary data was anMSL/Orthometric heights determined on survey stations built in the study area and the ellipsoidal heights of the same points measured differentially using a combined GNSS (GPS/GLONASS) Trimbledual frequency receivers in the Static Positioning mode. The Orthometric height was determined trigonometrically using a set of Total Station (Sokkia Set 330) equipment. Stations were spaced about 1kilometre apart along the routes described in the study area and within the area surrounded by the routes.

### 2.2 DATA PROCESSING

In all set-ups, instrument and target heights were measured aside the slope distances, horizontal distances, and vertical angles at each station with respect to the trigonometric heightening. Differential observations using the Trimble dual frequency combined GNSS(GPS/GLONASS) receivers were made on all the stations. The geoid-ellipsoid separation/undulation at each station was estimated using EGM2008 geodetic model calculator software with the WGS84 ellipsoidal coordinates of each station as input.

### 2.3 ANALYSIS OF DATA
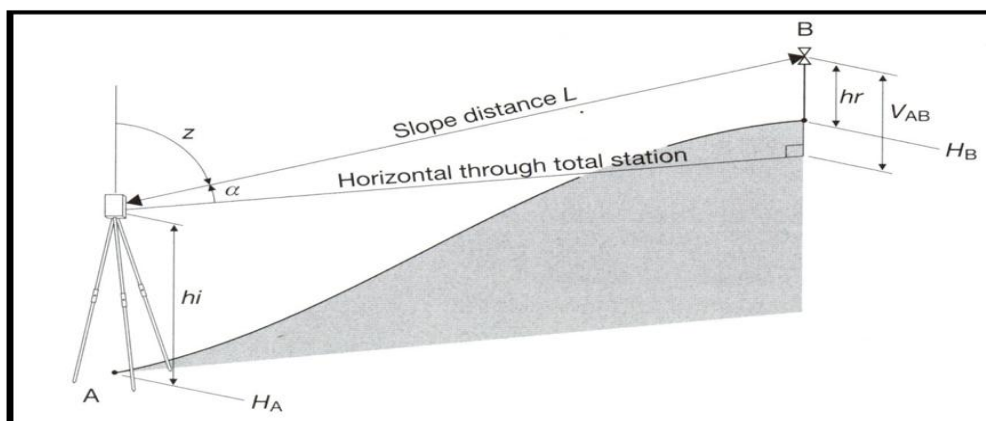
Figure 3: Principles of Trigonometric leveling (source: Surveying for Engineers by Uren and Price).

The slope distance L and vertical angle α between A and B were measured together with the height of the instrument hi above A and the height of the reflector hrabove B. Thus;

$HB$ = HA + hi + L Sin a - hr ...................................................... (2.1)

The total misclosure for the trigonometric heightening was constrained to 1st order acceptability, defined as;

$\text{Total misclosure} \leq 0.5 * \sqrt{M} \text{ (mm)},$ Where M = total distance in kilometres (km).

Orthometric heights (Hp) from the GPS heights were calculated by subtracting geoid-ellipsoid separation (N) from the ellipsoidal heights (hp) using the following equationas illustrated in figure 1:

Hp = hp - Np ........................................................................(2.2)

Np was estimated using the software, Alltrans EGM2008 Calculator 1.2 ( EGM 2008 geodetic model calculator).This Calculator was loaded with EGM2008 file named as: (Und_min10x10_egm2008_isw=82_WGS84_TideFree_SE) downloaded from the website of National Geospatial Intelligence Agency. This EGM 2008 file is 10'x10' grid model and with the WGS84 geographical coordinates as input the geoid undulation was estimated by the Bi-Quadratic Approach of Interpolation.

The Orthometric heights from trigonometric leveling were compared with those orthometric heights reduced from GPS observations by computing the residuals. The Orthometricheights from the trigonometric leveling were used as the standard (model).

R = H$_{observed}$ - H$_{model}$ ................................................................(2.3)

Where:

$R$ = residual at each point

$H_{observed}$ = actual orthometric heights reduced from GPS observation

$H_{model}$ = orthometric heights reduced from trigometrical leveling

The accuracy estimation was based on the criteria of the root mean square error RMSE, which was computed as:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(X_{obs,i} - X_{model,i}\right)^2}{n}}$$ .......................................................................................(2.4)

Where $X_{obs}$ is observed values and $X_{model}$ is modeled values at point $i$.

Pearson Product-Moment Coefficient of Correlation was used to predict the relationship between the two height systems:

$$r = \frac{n\left(\sum xy\right) - \left(\sum x\right)\left(\sum y\right)}{\sqrt{\left[\left[n\sum x^2 - \left(\sum x\right)^2\right]\left[n\sum y^2 - \left(\sum y\right)^2\right]\right]}}$$ ...........................................................................................(2.5)

Where       x = Orthometric heights determined by GPS

y = Orthometric heights determined by trigonometricleveling(MSL heights)

[1] noraspinto@yahoo.co.uk

Otoo-Kwofie, C. (2015) Assessing The Reliability Of Dual Frequency Differential GNSS Observations For Orthometric Height Determination In Parts Of Accra, Ghana. In: Mojekwu, J.N., Ogunsumi,L.O., Ojigi,L.M. Atepor, L., Thwala,D.W., Sackey,S. Awere E., and Bamfo-Agyei, E. (Eds) African Journal of Applied Research .(AJAR) Journal, Vol.1,No.1 ISSN 2408-7920 January 2015, Cape Coast, Ghana. 30-45

n= number of stations

$r$ = correlation co-efficient.

The Correlation co-efficient "$r$" was used to estimate the quality of fit of MSL heights to the GPS-derived Orthometric heights. Finally a Linear Regression Analysis was performed on the data to compare the relationship between the two height systems. Below is the mathematical model of the simple linear regression analysis:

$$y_i = a + bx_i + e_i \quad \text{................................................................(2.6)}$$

$$\text{Where, } b = \frac{\sum_{i=1}^{n} x_i y_i - n\overline{x}\,\overline{y}}{\sum_{i=1}^{n} x_i^2 - n\overline{x}^2} \quad \text{................................................................(2.7)}$$

$$a = \overline{y} - b\overline{x} \quad \text{................................................................(2.8)}$$

$\overline{x}$ = Mean of the predictor variable (i.e. orthometric heights determined by GPS)

$\overline{y}$ = Mean of the response variable (i.e. orthometric heights determined by leveling)

$a$ = Constant coefficient or intercept

$b$ = Slope

$e$ = Random disturbance or error, which measures the discrepancy in the approximation of $y_i$

$i$ = Station numbers

The linear regression analysis was also used to deduce $e$ which is the least squares residuals as a measure of random disturbances or errors.

## 3.0 RESULTS AND DISCUSSIONS
### 3.1 Results of The Trigonometric Leveling
Table 3.1 presents the Reduced Levels of the permanent concrete Type C beacons of the trigonometric leveling. Among the 35 points heighted, 10 of them were permanent concrete Type C beacons and 25 were temporary marks drawn on the ground with black paint to facilitate the leveling of the permanent beacons. The permanent beacons were numbered serially from KAAF/GE/11/12/2014/1 to KAAF/GE/11/12/2014/10. The existing vertical control numbered GA TP3 was used as the Benchmark for the exercise. The Reduced Level of the Benchmark (TP3) used was 51.6084m.

*Table 3.1: The Reduced Levels (MSL) from the Trigonometric Leveling*

| Stations | RL/m | Adjusted RL/m |
|---|---|---|
| KAAF/GE 11/12/14/1 | 42.1720 | 42.1681 |
| KAAF/GE 11/12/14/2 | 13.8622 | 13.8524 |
| KAAF/GE 11/12/14/3 | 40.2854 | 40.2737 |
| KAAF/GE 11/12/14/4 | 26.2907 | 26.2770 |
| KAAF/GE 11/12/14/5 | 38.2902 | 38.2736 |
| KAAF/GE 11/12/14/6 | 40.4335 | 40.4130 |
| KAAF/GE 11/12/14/7 | 35.0350 | 35.0135 |
| KAAF/GE 11/12/14/8 | 55.6642 | 55.6618 |
| KAAF/GE 11/12/14/9 | 24.3043 | 24.2945 |
| KAAF/GE 11/12/14/10 | 37.7162 | 37.6908 |
| TP3 | 51.6377 | 51.6084 |

## 3.2 Results of GNSS (Gps/Glonass) Differential Observations

The Trimble dual frequency Combined GPS/GLONASS (R4) receivers were used for the differential observations. The occupation time at each station was 20 minutes. The coordinates and the heights of the TypeC beacons listed in Appendix A were determined differentially with the control station SGGA SS 11/96/1 as reference station and SGGA EX G659/13/7 as check station. The height of each rover set-up was 2.00 m and was maintained throughout the observation. Subsequent computations can be found in Appendix A.

### 3.3 Linear Regression Analysis

*Table 3.2: Linear Regression Analysis*

| STATIONS | Hgps | Ht | Hgps*Ht | Hgps$^2$ | Predicted /fitted Ht | Least Squares residuals, (e) = Ht - predicted Ht |
|---|---|---|---|---|---|---|
| KAAF/GE/11/12/14/1 | 43.6046 | 42.1681 | 1838.7231 | 1901.361141 | 43.2896 | - 1.1215 |
| KAAF/GE/11/12/14/2 | 13.8752 | 13.8524 | 192.2048 | 192.521175 | 13.7861 | 0.0663 |
| KAAF/GE/11/12/14/3 | 39.7712 | 40.2737 | 1601.7334 | 1581.748349 | 39.4853 | 0.7884 |
| KAAF/GE/11/12/14/4 | 25.9615 | 26.2770 | 682.1903 | 673.9994823 | 25.7806 | 0.4964 |
| KAAF/GE/11/12/14/5 | 39.3444 | 38.2736 | 1505.8518 | 1547.981811 | 39.0618 | - 0.7882 |
| KAAF/GE/11/12/14/6 | 40.1163 | 40.4130 | 1621.2200 | 1609.317526 | 39.8278 | 0.5852 |
| KAAF/GE/11/12/14/7 | 35.6711 | 35.0135 | 1248.9701 | 1272.427375 | 35.4164 | - 0.4029 |
| KAAF/GE/11/12/14/8 | 55.2188 | 55.6618 | 3073.5778 | 3049.115873 | 54.8155 | 0.8463 |
| KAAF/GE/11/12/14/9 | 24.2591 | 24.2945 | 589.3627 | 588.5039328 | 24.0911 | 0.2034 |
| KAAF/GE/11/12/14/10 | 38.6396 | 37.6908 | 1456.3574 | 1493.018688 | 38.3623 | - 0.6715 |

SUMMATION ($\sum$)       356.4618   353.9184   13810.1915   13909.99535

where,Hgps are Orthometric heights obtained from GNSS (GPS/GLONASS) observations and Ht are Orthometric heights obtained through trigonometric observations.

Mean of Hgps       =       356.4618 / 10
=       35.6462

Mean of Ht       =       353.9184 / 10
=       35.3918

From equations (2.7) and (2.8), the constant/intercept of the regression(i.e. *a*) and coefficients of the Hgps (i.e.*b*) are calculated as shown below:

$$b = \frac{13810.1915 - 10 * 35.6462 * 35.3918}{13909.99535 - 10 * 35.6462^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.9)$$
$$= \quad 0.9924$$

$$a = 35.3918 - 0.9924 * 35.6462 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (2.10)$$
$$= \quad 0.0165$$

Therefore the mathematical model that resulted after the regression analysis is stated below as:

$$y_i = 0.0165 + 0.9924x_i + e_i \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (2.11)$$

NB: the fitted/predicted Ht values and the least squares residuals e in the tables were calculated using the equations below:

$$fitted/predicted\ Ht_i = 0.0165 + 0.9924 * Hgps_i \dots \dots \dots \dots \dots \dots \dots \dots (2.12)$$

$$e_i = Ht - fitted/predicted\ Ht_i \dots \dots (2.13)$$

## 3.4 Summary of The Results

The series of analysis performed on the data revealed the following findings:

1. The total misclosure committed in the trigonometrical leveling was +0.0293 m, with the misclosure per each instrument station estimated as +0.0010 m. The total chainage was about 13 km with 35 instrument set-ups/stations.
2. The acceptable errors per height allowed for such first order vertical control surveys was estimated as 1.8163 mm.
3. The estimated Geoidal undulations range from +24.1589m to +24.3376m, with an average undulation value of 24.2659m.
4. The residuals computed arithmetically between the GPS/GLONASS-derived Orthometric Heights and the MSL heights determined trigonometrically ranges between -0.5025m and +1.4365m, with an average residual of 0.5730 m.
5. The Root Mean Square Error was estimated as 0.7198m
6. The Correlation Coefficient estimated between the GPS/GLONASS-derived Orthometric Height and MSL heights determined trigonometrically was +0.9981.
7. The Linear Regression Analysis performed resulted with the constant/intercept of the regression, $a$ and the coefficients of the predictor variable (i.e. the GPS/GLONASS-derived Orthometric heights), $b$ are:
   a           =           + 0.0165
   b           =           + 0.9924
8. The linear regression model that resulted is stated below:

$$y_i = 0.0165 + 0.9924x_i + e_i \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (2.14)$$

9. The least squares residuals that resulted from the regression analysis ranges from -1.1215 to +0.8463, with an average residual, in absolute terms, of 0.5970m.

## 3.5 DISCUSSIONS

The Correlation Coefficient implies that the Orthometric Heights determined from Trigonometric leveling using Total Station and those from GNSS(GPS/GLONASS) derived

differential observations are positively and strongly correlated. Secondly, based on the correlation coefficient it is clear that the quality of fit of MSL heights to the GPS-derived orthometric heights is very high.

The least squares residuals that resulted from the regression analysis ranges from - 1.1215m to +0.8463m, with an average residual, in absolute terms, of 0.5970m also implying a more quality fit of the model to the data. The least squares residuals evaluate the random disturbances or errors, which measure the discrepancy in the approximation to Ht.

## 4.0.    CONCLUSION AND RECOMMENDATIONS

The study usedsimple linear regression analysis, root mean square errors, calculation of residuals and scatter-plot graphs to evaluate the reliability of differential observations made with GNSS(GPS/GLONASS) dual frequency receivers by comparing it with MSL heights determined  trigonometrically using Total Station.

The adjusted MSL heights determined trigonometrically using the Total Station ranged between 13.8524 m to55.6618m with a total/cumulative misclosure of +0.0293m. The GNSS (GPS/GLONASS) Ellipsoidal heights observed was between 38.1460m and 79.4330m with a total observed error of 0.15m. The Geoidal undulations estimated with the EGM2008 calculator using the Bi-quadratic method is between +24.1589m to +24.3376m.

The analysis revealed that theGNSS (GPS/GLONASS) derived Orthometric heights were reliable and comparable to the MSL heights determined trigonometrically. The RMSE was estimated as 0.7198m. The Correlation Coefficient was +0.9981 denoting a measure of higher quality of fit. The average least squares residuals from the linear regression analysis was estimated in absolute terms, as 0.5970m. The resulting regression model has a slope of +0.9924 and an intercept of +0.0165. In the aggregate, the results of the analysis confirmed by the scatter-plot, show a veryhigh quality of fit of the model to the data.

In view of the above, the team wishes to conclude that thecombined GPS/GLONASS-derived Orthometric heights are reliable and the results comparable to the MSL heights determined trigonometrically.

It is recommended that the Engineers and Surveyors in Ghana adopt fully GNSS receivers in height determination. In addition to this, the determination of the National Geoid Model by Ghana Land Administration project should be seriously pursued since it is a major breakthrough in height determination.

## REFERENCES

Abd-Elmotaal, H. A. (2008). *Evaluation of the EGM2008 Geopotential Model for Egypt.*Minia University, Civil Engineering Department, Faculty of Engineering.Minia, Egypt: Minia University.

Ayer, J., andOfosu, C. (2006).Map Coordinate Referencing and the Use of GPS Datasets in Ghana.*Journal of Science and Technology, Vol.28* (No.1), pg 9 - 12.

[1] noraspinto@yahoo.co.uk

Otoo-Kwofie, C. (2015) Assessing The Reliability Of Dual Frequency Differential GNSS Observations For Orthometric Height Determination In Parts Of Accra, Ghana. In: Mojekwu, J.N., Ogunsumi,L.O., Ojigi,L.M. Atepor, L., Thwala,D.W., Sackey,S. Awere E., and Bamfo-Agyei, E. (Eds) African Journal of Applied Research .(AJAR) Journal, Vol.1,No.1 ISSN 2408-7920 January 2015, Cape Coast, Ghana. 30-45

Azab, M., Shoukry, M. N., El-Rabbany, A., and Khalil, R. (2011). Precise Point Positioning Using Combined GPS/GLONASS Measurements. *FIG Working Week 2011* , pp. 1 - 8.

Bannister, A., and Raymond, S. (1995). *Surveying.* Essex, England: Longman Group Limited.

Ghana Statistical Service.(2010). 2010 Population and Housing Census.

Ghana Survey Records. (1936). *Technical Instruction for the Cadastral Branch* (2nd Edition ed.). Gold Coast (now Ghana): Government Printing Department.

Henriques, M. J., andCasaca, J. (2001).Monitoring vertical displacements by means of geometric levelling.In *Historical Constructions* (pp. 403 - 412).Lisborn, Portugal: The Applied Geodesy Division, Dams
Department of The National Laboratory for Civil Engineering (LNEC).

Mohammed, N. Z., Mohammed, A. E., andBakry, O. (2012).Evaluation of GPS in Orthometric Heights Determination in Khartoum State (Sudan).*International Journal of Multidisciplinary Sciences and Engineering, Vol. 3* (No. 8), pp.1-5.

Survey of Ghana.(2008). *Technical Guidelines for Spatial Data Capture and Presentation in Ghana.* Accra, Ghana: Survey & Mapping Division, Lands Commission.

Uren, J., and Price, W. F. (2006).*Surveying for Engineers* (4th Edition ed.). London: Palgrave Macmillan.

## APPENDIX A

*Table 4.2: WGS84 Geodetic Coordinates, observed Ellipsoidal Heights, Estimated Geoidal Separations Using EGM 2008and computed Orthometric heights from Ellipsoidal Heights of type C beacons.*

| STATIONS | LAT/N | | | LONG/W | | | ELLIPSOIDAL HEIGHTS/m (H) | GEOIDAL SEPARATION/m (N) | ESTIMATED ORTHOMETRIC HEIGHT/m h = H - N |
|---|---|---|---|---|---|---|---|---|---|
| | DEG | MINS | SEC | DEG | MINS | SEC | | | |
| KAAF/GE/11/12/14/1 | 5 | 35 | 55.29 | 0 | 11 | 22.24 | 67.816 | 24.211 | 43.605 |
| KAAF/GE/11/12/14/2 | 5 | 36 | 26.88 | 0 | 12 | 9.15 | 38.146 | 24.271 | 13.875 |
| KAAF/GE/11/12/14/3 | 5 | 37 | 2.95 | 0 | 12 | 47.14 | 64.104 | 24.333 | 39.771 |
| KAAF/GE/11/12/14/4 | 5 | 37 | 10.96 | 0 | 11 | 51.22 | 50.290 | 24.329 | 25.962 |
| KAAF/GE/11/12/14/5 | 5 | 37 | 31.84 | 0 | 10 | 35.03 | 63.682 | 24.338 | 39.344 |
| KAAF/GE/11/12/14/6 | 5 | 37 | 3.66 | 0 | 10 | 32.19 | 64.413 | 24.297 | 40.116 |
| KAAF/GE/11/12/14/7 | 5 | 37 | 3.58 | 0 | 10 | 25.60 | 59.966 | 24.295 | 35.671 |
| KAAF/GE/11/12/14/8 | 5 | 36 | 7.09 | 0 | 10 | 31.23 | 79.433 | 24.214 | 55.219 |
| KAAF/GE/11/12/14/9 | 5 | 35 | 29.98 | 0 | 10 | 30.61 | 48.418 | 24.159 | 24.259 |
| KAAF/GE/11/12/14/10 | 5 | 36 | 2.59 | 0 | 10 | 52.02 | 62.853 | 24.213 | 38.640 |
| SGGA SS 11/96/1 | 5 | 35 | 36.29 | 0 | 10 | 55.87 | 75.833 | | |
| SGGA EX G659/13/7 | 5 | 35 | 29.42 | 0 | 10 | 29.30 | 75.368 | | |

[1] noraspinto@yahoo.co.uk

*Table 4.3: Computation of the Root Mean Square Error and Correlation Coefficient*

| STATIONS | ORTHOMETRIC HEIGHTS FROM GPS OBSERVATIONS (Hgps) | ORTHOMETRIC HEIGHTS FROM TRIG. LEVELLING (Ht) | RESIDUALS (R) $R = Hgps - Ht$ | RESIDUALS SQUARED ($R^2$) | Hgps * Ht | $(Hgps)^2$ | $(Ht)^2$ |
|---|---|---|---|---|---|---|---|
| KAAF/GE/11/12/14/1 | 43.605 | 42.168 | 1.437 | 2.064 | 1838.723 | 1901.361 | 1778.149 |
| KAAF/GE/11/12/14/2 | 13.875 | 13.852 | 0.023 | 0.001 | 192.205 | 192.521 | 191.889 |
| KAAF/GE/11/12/14/3 | 39.771 | 40.274 | -0.503 | 0.253 | 1601.733 | 1581.748 | 1621.971 |
| KAAF/GE/11/12/14/4 | 25.962 | 26.277 | -0.316 | 0.099 | 682.190 | 673.999 | 690.481 |
| KAAF/GE/11/12/14/5 | 39.344 | 38.274 | 1.071 | 1.147 | 1505.852 | 1547.982 | 1464.868 |
| KAAF/GE/11/12/14/6 | 40.116 | 40.413 | -0.297 | 0.088 | 1621.220 | 1609.318 | 1633.211 |
| KAAF/GE/11/12/14/7 | 35.671 | 35.014 | 0.658 | 0.432 | 1248.970 | 1272.427 | 1225.945 |
| KAAF/GE/11/12/14/8 | 55.219 | 55.662 | -0.443 | 0.196 | 3073.578 | 3049.116 | 3098.236 |
| KAAF/GE/11/12/14/9 | 24.259 | 24.295 | -0.035 | 0.001 | 589.363 | 588.504 | 590.223 |
| KAAF/GE/11/12/14/10 | 38.640 | 37.691 | 0.949 | 0.900 | 1456.357 | 1493.019 | 1420.596 |
| | $\Sigma = 356.4618$ | $\Sigma = 353.9184$ | | $\Sigma = 5.181$ | $\Sigma = 13810.192$ | $\Sigma = 13909.995$ | $\Sigma = 13715.569$ |

$$RMSE = \sqrt{\frac{5.18090348}{10}}$$

[1] noraspinto@yahoo.co.uk

$$= 0.7198$$

$$\text{Correlation Coefficient}, r = \frac{(10 * 13810.1915) - 356.4618 * 353.9184}{\sqrt{[(10 * 13909.99535 - 356.4618^2)(10 * 13715.56861 - 353.9184^{\,2})]}}$$

$$= +0.998$$